Surface Impedance of a Clean Two-Band Superconductor near H_{c2}

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The surface impedance is calculated for two-band superconductors in the mixed state near H_{c2}. The electromagnetic response function for the two-band system is seen to be the sum of the electromagnetic response in the individual bands. The individual response functions are then evaluated using a technique developed by Caroli and Maki. The resulting two-band expression for the surface impedance in the mixed state near H_{c2} is used to analyze surface-impedance measurements on niobium. Both the purity dependence and the anisotropy in the surface impedances observed by Hibler and Maxwell can be accounted for qualitatively by the two-band expression.

I. INTRODUCTION

In a recent paper,[1] Cyrot and Maki showed that for pure type-II superconductors ($l \gg \xi_0$, where l is the electronic mean free path and ξ_0 is the coherence length) in high fields, the perturbation expansion in powers of the order parameter leads to unphysical results. In order to circumvent this difficulty, Maki[2,3] made the conjecture that the effect of the magnetic field on the order parameter of a pure type-II superconductor in high fields is similar to that of a transport current. He was then able to obtain explicit expressions for the ultrasonic attenuation[2] and thermal conductivity[3] in the mixed state of a pure type-II superconductor near its upper critical field. Using the same approach, Caroli and Maki[4] have calculated the electromagnetic response for the pure superconductor in high fields. An interesting feature of all the transport coefficients obtained by Kagiwada *et al.*[5] for the mixed state of pure type-II superconductors near H_{c2} is the prediction of a $(H_{c2}-H)^{1/2}$ field dependence instead of the linear dependence observed in superconducting alloys.

Since pure niobium is an intrinsic type-II superconductor, measurements of the various transport properties[5–10] have been taken in order to verify the correctness of the various coefficients obtained by Kagiwada *et al.* Qualitative agreement with the theoretical expressions has been seen in the ultrasonic attenuation[5–7] and thermal conductivity[8,9] data for niobium near H_{c2}; i.e., the $(H_{c2}-H)^{1/2}$ field dependence is clearly observed. Quantitative agreement between the experimental data and the theoretical expressions can be achieved only by treating the density of states as an adjustable parameter. However, this would imply a purity dependence in the density of states. This purity dependence in the density of states would lead to an unobserved purity dependence in the magnetic properties.[6,7,9]

Recent calculation of the band structure in niobium[11] has shown the Fermi surface to be comprised of three sheets which overlap in certain directions. The electrons on one of the sheets are identified as being s electrons, while those on the other two sheets belong to the d band. To account for superconductivity in transition elements which have similar band structures, a two-band model was proposed by Suhl, Matthias, and Walker (SMW).[12] Using this model, Sung and Shen[13] and Radhakrishnan[14] were able to obtain numerical fits of the experimental data for various properties of niobium. More recently, direct evidence for the two-band model has been obtained in the discovery of a second transition temperature[15] and a second energy gap[16] in niobium. Therefore, it should be expected that any theoretical expressions for the various transport properties of superconducting niobium will reflect the two-band nature of niobium.

It is the purpose of this paper to obtain the two-band expression for the surface impedance of transition-element superconductors in high fields and compare it with the experimentally measured surface impedance of niobium[10] in the mixed state. Previously, the author obtained the two-band expressions for the ultrasonic attenuation and thermal conductivity in transition-metal superconductors in the mixed state.[17] Using the two-band expressions, it was possible to explain the purity dependence seen in the experimental measurements[6–9] without implying the contradictory purity dependence in the magnetic properties. We shall see that the purity dependence in the surface impedance observed by Hibler and Maxwell[10] (HM) can be explained by the two-band expression. More importantly, the observed anisotropy in the surface impedance, which is in total disagreement with the anisotropy predicted by the one-band expression of Caroli and Maki, can be explained qualitatively with the two-band expression.

II. ELECTROMAGNETIC RESPONSE

An important point to remember when calculating the electromagnetic response for the two-band system is that the electric current in the two-band system is just the sum of the electric currents in the individual bands. This leads to a response function of the form

$$Q_{\mu\mu}(q,\omega)=\langle[j_{s\mu},j_{s\mu}]\rangle_{qq\omega}+\langle[j_{d\mu},j_{d\mu}]\rangle_{qq\omega}, \quad (2.1)$$

where $j_{s(d)\mu}$ is the μth component of the electric current in the $s(d)$ band and $\omega=(2nl)k_BT$. In order to evaluate the individual correlation functions appearing in (2.1), it will be necessary to make the conjecture that the effects of the magnetic field on the energy gaps in the individual bands are similar to those of a transport current in each band. With this assumption, the individual correlation functions can be evaluated in exactly the same manner as in Caroli and Maki[4] (in the system where $\hbar=k_B=c=1$),

$$\langle[j_{s(d)\mu},j_{s(d)\mu}]\rangle_{qq\omega}=-\frac{N_{s(d)}e^2}{m_{s(d)}}\left[3\pi T\sum\int\frac{d\Omega}{4\pi}\chi_\mu^2\times\int d\alpha\,\rho_{s(d)}(\alpha,\Omega)\frac{\omega_n}{|\omega_n|}\frac{\Delta_{s(d)}^2}{(\omega_n-i\alpha)^3}-\frac{3\pi\omega i}{4qv_{s(d)F}}+\frac{\pi i\omega\Delta_{s(d)}}{2T}\left(1-\ln\frac{8\Delta_{s(d)}}{|\omega|}\right)\int\frac{d\Omega}{4\pi}3\chi_\mu^2\delta(v_{s(d)F}q)\times\int_{-\infty}^{\infty}d\alpha\,\rho_{s(d)}(\alpha,\Omega)\cosh^{-2}\frac{\alpha}{2T}\right],\quad(2.2)$$

where

$$\rho_{s(d)}(\alpha,\Omega)=(1/\pi^{1/2}\epsilon_{s(d)}\sin\theta)\exp[-(\alpha/\epsilon_{s(d)}\sin\theta)^2],\quad(2.3)$$

with

$$\epsilon_{s(d)}=v_{s(d)F}(\tfrac{1}{2}eH_{c2})^{1/2},\quad(2.4)$$

and where $m_{s(d)}$ is the mass of a $s(d)$ electron, $N_{s(d)}$ is the density of states in the $s(d)$ band, $v_{s(d)F}$ is the velocity of a $s(d)$ electron at the Fermi surface, χ_μ is the direction cosine, and θ is the angle between the magnetic field and the normal to the surface of the sample. The second term on the right-hand side of (2.2) is the anomalous skin-effect expression in the normal state.

Adding the correlation functions associated with the two bands together, we have for the real and imaginary parts

$$\mathrm{Re}Q_{\mu\mu}(q,\omega)=-\frac{N_se^2}{m_s}\frac{3\Delta_s^2}{(2\pi T)^2}\int_{-\infty}^{\infty}d\alpha\int\frac{d\Omega}{4\pi}\times\cos^2\theta\,\rho_s(\alpha,\Omega)\psi^{(2)}\left(\frac{1}{2}-\frac{i\alpha}{2\pi T}\right)-\frac{N_de^2}{m_d}\frac{3\Delta_d^2}{(2\pi T)^2}\times\int_{-\infty}^{\infty}d\alpha\int\frac{d\Omega}{4\pi}\cos^2\theta\,\rho_b(\alpha,\Omega)\psi^{(2)}\left(\frac{1}{2}-\frac{i\alpha}{2\pi T}\right),\quad(2.5)$$

$$\mathrm{Im}Q_{\mu\mu}(q,\omega)=\frac{3\pi N_se^2\omega\tau_s}{4m_sql_s}\times\left[1-\frac{\Delta_s}{2T}\left(1-\ln\frac{8\Delta_s}{|\omega|}\right)G_s(\rho_s,\theta)\right]+\frac{3\pi N_de^2\omega\tau_d}{4m_dql_d}\times\left[1-\frac{\Delta_d}{2T}\left(1-\ln\frac{8\Delta_d}{|\omega|}\right)G_d(\rho_d,\theta)\right],\quad(2.6)$$

with

$$G_{s(d)}(\rho_{s(d)},\theta)=\frac{4}{\pi^{1/2}}\int_0^1dz\,[1-z^2]^{1/2}\times\int_{-\infty}^{\infty}dx\,\frac{\exp[-x^2/(1-z^2\sin^2\theta)]}{[1-z^2\sin^2\theta]^{1/2}}\cosh^{-2}(\pi\rho_{s(d)}x),\quad(2.7)$$

where $\psi^{(2)}$ is the tetra-γ function, $\tau_{s(d)}$ is the collision lifetime of a $s(d)$ electron, and $l_{s(d)}$ is the collision mean free path for a $s(d)$ electron. An identical expression for $\mathrm{Re}Q_{\mu\mu}(q,\omega)$ can be obtained by the direct expansion of the Green's function in powers of the energy gaps (the Green's functions for the different band electrons can be expressed as separate Gor'kov equations where the energy gaps play the role of the order parameters). However, the superconducting corrections to $\mathrm{Im}Q_{\mu\mu}$ cannot be obtained by the direct expansion. This singular behavior in Δ is a consequence of a coalescence of the two singularities in the density of states.[2]

In deriving the above equations, we have assumed that the microwave current is traveling parallel to the magnetic field, since it is in this geometry that the measurements[10] were taken. For $\theta=0$ and $\frac{1}{2}\pi$, the factors $G_{s(d)}(\rho,\theta)$ reduce to the functions $f^{(1)}(\rho)$ and $f^{(2)}(\rho)$ defined in Ref. 4, respectively. The complex conductivity is obtained from the relation

$$\sigma_s=\omega^{-1}\,\mathrm{Im}Q_{\mu\mu}(q,\omega).\quad(2.8)$$

The normalized surface impedance is given by

$$\frac{Z_s}{Z_n}=2\exp(i\pi/3)\left[1-\frac{N_s}{N_s+N_d}\frac{\Delta_s}{2T}\left(1-\ln\frac{8\Delta_s}{|\omega|}\right)G_s(\rho_s,\theta)-\frac{N_d}{N_s+N_d}\frac{\Delta_d}{2T}\left(1-\ln\frac{8\Delta_d}{|\omega|}\right)G_d(\rho_d,\theta)\right]^{-1/3},\quad(2.9)$$

where we have assumed that the Fermi surface can still be approximated by a sphere. This results in a systematic error being introduced into the expression, but it should not affect the qualitative understanding of the purity dependence and anisotropy seen in the surface impedance.

For the purpose of analyzing the experimental data of HM, we rewrite the surface impedance in the form

$$\frac{Z_s}{Z_n}=2\exp(i\pi/3)\left\{1-\frac{\Delta_d}{2T}\left(1-\ln\frac{8\Delta_d}{|\omega|}\right)G_d(\rho_d,\theta)+\frac{N_s}{N_s+N_d}\left[\frac{\Delta_d}{2T}\left(1-\ln\frac{8\Delta_d}{|\omega|}\right)G_d(\rho_d,\theta)-\frac{\Delta_s}{2T}\left(1-\ln\frac{8\Delta_s}{|\omega|}\right)G_s(\rho_s,\theta)\right]\right\}^{-1/3}.\quad(2.10)$$

In this form, we see immediately that the two-band expression reduces to the result of Caroli and Maki when the s band disappears.

III. PURITY DEPENDENCE

Nonmagnetic impurities affect the one- and two-band model in quite different ways. In the one-band superconductors, the major effect of the impurities is to smear out the Fermi surface and thus diminish the anisotropy in the superconductors. In the two-band model, the dominant effect of the impurities at low temperatures[18] is to cause interband transitions. The rates at which the transition between the two bands occur are given by the transition matrix elements

$$\Gamma_{sd}=\pi n_i N_d \int (d\Omega/4\pi)\ |V_{sd}(\theta)|^2 \tag{3.1}$$

and

$$\Gamma_{ds}=\pi n_i N_s \int (d\Omega/4\pi)\ |V_{sd}(\theta)|^2, \tag{3.2}$$

where Γ_{sd} is the transition matrix for the scattering of a s electron into the d band, Γ_{ds} is the matrix for the scattering of the d electrons into the s band, n_i is the impurity concentration, and $V_{sd}(\theta)$ is the interband scattering potential. Comparing the two transition matrix elements, we see that $\Gamma_{sd}/\Gamma_{ds}\approx N_d/N_s$. For concentrations of impurities such that Γ_{sd} is comparable to the energy gap of the lower band, Wong and Sung[19] have shown that only the density of states for the s electrons is affected by the impurity scattering. The d band is not affected much by the presence of the impurities.

Since the thermodynamical properties of the two-band superconductors are dominated by the more densely populated d band, the change in the impurity concentration will not affect the magnetic properties. A purity dependence does exist in the upper critical field and the Ginsburg-Landau parameter since these properties depend to some extent on the s electrons. By taking into account the contributions due to the s electrons, Sung[20] was able to explain the purity dependence in these properties.

The need for a model in which a purity dependence occurs in the density of states without implying a similar dependence in the magnetic properties is seen in the surface impedance measurements on niobium in the mixed state near H_{c2}.[10] HM performed their measurements on two extremely pure niobium samples with residual resistivity ratios of 6500 and 11 000. Over this range of purities, one expects the purity dependence of the order parameter to be only a few percent. By introducing an adjustable parameter into the expression for the order parameter, HM attempted to fit numerically the experimental data to the expression obtained by Caroli and Maki.[4] The form used by HM for the order parameter was

$$\Delta_d^2=\frac{\alpha}{2N_d\pi}\frac{H_{c2}-H}{1.16[2\kappa_2^2(t)-1]}\left(H_{c2}-\tfrac{1}{2}T\frac{dH_{c2}}{dT}\right), \tag{3.3}$$

where κ_2 is the second Ginzburg-Landau parameter, $H_{c2}-\frac{1}{2}T(dH_{c2}/dt)$ is obtained from the results of McConville and Serin,[21] and N_d has the value 5.6×10^{34} states/cm^3 erg (the value obtained from the specific-heat measurements on niobium[22]). The procedure used by HM is equivalent to treating the density of states as an adjustable parameter.

To obtain the fit of the experimental data on the RRR 6500 sample, the value for α was found to be 2.74. For the fit of the data on the RRR 11 000 sample, α was found to be 1.38. Direct comparison with the work of Kagiwada *et al.*[5] on the ultrasonic attenuation in niobium is not possible since different values for the Fermi velocity were used in the two works. However, the larger values of α required to fit the data on the surface impedance for the less pure specimens are consistent with the observations of Kagiwada *et al.* This apparent purity dependence in the surface impedance is inconsistent with the observed behavior of the order parameter, i.e., the order parameter does not vary as much as the implied variation in the density of states.

To see that the two-band expression (2.10) can explain qualitatively the purity dependence, we note that the ratio $N_s/(N_s+N_d)$ increases with the purity of the specimens. The leading correction to the surface impedance caused by the presence of a second band is

$$-\frac{N_s}{N_s+N_d}\left(\frac{\Delta_d}{2T}\ln\frac{8\Delta_d}{|\omega|}G_b(\rho_b,\theta)-\frac{\Delta_s}{2T}\ln\frac{8\Delta_s}{|\omega|}G_s(\rho_s,\theta)\right), \tag{3.4}$$

which becomes smaller as specimens become less pure. Thus we see the normalized surface impedance of the mixed states near H_{c2} increase as the ratio $N_s/(N_s+N_d)$ increases, i.e., the surface impedance is higher in the purer specimen. Quantitative comparison of the experimental data with the two-band expression can only be made if the density-of-states ratios for the two specimens were known.

The advantage of using the two-band expression for qualitative comparison is that one is not led to false purity dependence in the order parameter.

IV. ANISOTROPY

In addition to the purity dependence of the surface impedance in the mixed state of niobium near H_{c2}, HM[10] observed an anisotropy in the surface impedence as the angle between the magnetic field and the normal to the surface was increased. They found the surface impedance grew larger as the angle increased. At an angle of 50°, the increase in the normalized impedance was over 7%. This increasing impedance is in total disagreement with the one-band model which predicts a decreasing surface impedance.

To see that the observed anisotropy can qualitatively be accounted for in the two-band model, we write the two-band expression for the surface impedance in the mixed state near H_{c2} normalized to the value of the two-band expression at $\theta=0°$:

$$\frac{Z_s/Z_n|_\theta}{Z_s/Z_n|_{\theta=0°}}=\left(\frac{1+(\Delta_d/2T)(\ln 8\Delta_d/|\omega|-1)G_d(\rho_d,0°)-[N_s/(N_s+N_d)]\times[(\Delta_d/2T)(\ln 8\Delta_e/|\omega|-1)G_d(\rho_d,0°)-(\Delta_s/2T)(\ln 8\Delta_s/|\omega|-1)G_s(\rho_s,0°)]}{1+(\Delta_e/2T)(\ln 8\Delta_d/|\omega|-1)G_d(\rho_d,\theta)-[N_s/(N_s+N_d)]\times[(\Delta_d/2T)(\ln 8\Delta_d/|\omega|-1)G_d(\rho_d,\theta)-(\Delta_s/2T)(\ln 8\Delta_s/|\omega|-1)G_s(\rho_s,\theta)]}\right)^{1/3}.$$

We note that as the ratio $N_s/(N_s+N_d)$ goes to zero, the above ratio reduces to the ratio obtained by Caroli and Maki.[4] Using this fact, we can establish the inequality $G_d(\rho_d,\theta=0°)<G_d(\rho_d,\theta)$, where the inequality becomes larger as the angle increases. With the inequality, we have

$$\left[\frac{\Delta_d}{2T}\left(\ln\frac{8\Delta_d}{|\omega|}-1\right)G_d(\rho_d,\theta)-\frac{\Delta_s}{2T}\left(\ln\frac{8\Delta_s}{|\omega|}-1\right)G_s(\rho_s,\theta)\right]$$
$$>\left[\frac{\Delta_d}{2T}\left(\ln\frac{8\Delta_d}{|\omega|}-1\right)G_d(\rho_d,0°)-\frac{\Delta_s}{2T}\left(\ln\frac{8\Delta_s}{|\omega|}-1\right)G_s(\rho_s,0°)\right].$$

We can see that this last inequality can lead to the numerator being larger than the denominator. This could explain the increase in the surface impedance as the angle increases. Because the $N_s/(N_s+N_d)$ factor in the two-band expression for the anisotropy, we expect the anisotropy to depend on the purity of the specimens used. We believe that when dirty niobium samples are used, the anisotropy observed will be the decreasing surface impedance predicted by the one-band expression of Caroli and Maki.[4]

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